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Equivalent roughness for pressure drop calculations in mine ventilation

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ABSTRACT: The generalized procedure for the calculation of the pressure drop along tunnels is by using Atkinson's equation. The friction factor in Atkinson's equation is determined from measured or computed values of airway dimensions and tunnel interior finishing according to mining method and ground support type. This paper re-visits the friction factor according to Colebrook´s relationship and the Darcy-Weisbach equation, which are widely used in mechanical engineering. This method of calculating the friction has the advantage of a more reliable determination since the size of the airway is implicit in the method and only a representative absolute roughness has to be selected to determine the friction factor. Measurements of pressure drop performed at Codelco's El Teniente mine, Chile, permitted the determination of the absolute roughness for different tunnel sizes and wall finishing.

1 Introduction

In the technical literature for mine ventilation, the generalized procedure for the calculation of the pressure drop along tunnels is to derive the airway's resistance by using Atkinson's equation. This resistance is determined from measured or computed values of airway dimensions and the tunnel's interior finishing, including the type of ground support and mining method. Of all the factors in Atkinson's equation, the friction factor can be the most challenging to estimate. It has been documented (Blevins 1984) that friction factor can be related to fluid dynamic properties and the roughness of the tunnel walls to the diameter of the tunnel. This paper re-visits the friction factor according to Colebrook´s relationship and the Darcy-Weisbach equation, which are widely used in mechanical engineering, and by correlating the roughness with actual data measured for different tunnel sizes and wall finishing. This method of calculating friction factor has the advantage of a more reliable determination of the friction since the size of the airway is implicit in the method. An example problem is presented to show the impact of airway size on the calculation of friction factor.

2 Theoretical background

In modern fluid mechanics, the friction factor used to determine the pressure drop of a fluid flowing through ducts or airways is defined by a function of dimensionless parameters which characterizes it for different fluid properties, the size of the duct and the roughness of the inside surface.

The calculation of the pressure drop of the air flow according to the Darcy-Weisbach Equation (1) is defined as follows:

$$\Delta p = \left(\sum_i K_i + f_t \frac{L_t}{D_h} \right) \frac{\rho_a \mathrm{v}_\mathrm{m}^2}{2} \tag{1}$$

where Δp = Pressure drop (Pa)
K_i = Losses in tunnel
D_h = Hydraulic diameter (m)
f_t = Friction factor (dimensionless)
v_m = Air velocity (m/s)
L_t = Length of tunnel (m)
ρ_a = Air Density (kg/m^3)

If consistent units are used then the result is defined by the velocity head $\frac{\rho_a \mathrm{v}_\mathrm{m}^2}{2}$ (Pa). The air velocity corresponds to the mean airflow divided by the cross sectional area and is obtained from Equation (2).

$$\mathrm{v}_\mathrm{m} = \frac{Q}{A_t} \tag{2}$$

where Q = Average airflow (m^3/s)
A_t = Tunnel area (m^2).

The hydraulic diameter is a characteristic dimension representative of the tunnel and depends of the area and perimeter and is shown in Equation (3). The use of it simplifies the analysis and gives good results for typical cross sections of mine tunnels and shafts.

$$D_h = \frac{4A_t}{P_t} \tag{3}$$

where P_t = "wetted" perimeter of Tunnel area (m).

However, for more accurate analyses, an improved equivalent diameter calculation should be used for turbulent flow (Blevins, 1984). This is not an equivalent diameter of a circular section from which the area could be calculated; rather the area must be obtained from the geometry of the tunnel section. The "wetted" perimeter of Equation (3) in this case generally corresponds to the contour of the tunnel section since the air "floods" all the area and is a parameter which characterizes the wall friction.

The singular loss factors K_i of Equation (1) corresponds to a multiplier of the velocity head to account for the pressure drop of tunnel curves, inlets, exhaust, splits, joints, section transitions, etc. and are obtained from tables (for example see the work of Blevins (1984)). In Equation (1) each loss within the same tunnel section must have the same area. If transitions, or shock losses, occur in the tunnel, then the pressure drop through the tunnel must be divided into more sections in order to compute the various transition losses and ensure that each section's velocity head is accounted for in each calculation.

To obtain the friction factor, the Reynolds number, Re, and the relative roughness, ε_t, must be calculated according to Equations (4) & (5). Both numbers are dimensionless and characterize the flow from a similitude point of view, allowing the application of the model to different tunnel sizes, fluid properties and flow conditions.

$$\mathrm{Re} = \frac{\mathrm{v}_m D_h}{\nu_a} \tag{4}$$

$$\varepsilon_t = \frac{\mathrm{e}_t}{D_h} \tag{5}$$

where e_t = Tunnel wall absolute roughness (m)

ν_a = Kinematic viscosity of air (m^2/s)

as defined by $\nu_a = \frac{\mu_a}{\rho_a}$

where μ_a = Absolute viscosity (Pa·s)

In general, the absolute roughness of the tunnel wall is a characteristic value of the surface finish, but it does not correspond exactly to the roughness of the surface obtained according to the construction method used.

The friction factor is a function of the Reynolds number and relative roughness $f_t = \mathrm{F}(\mathrm{Re},\ e_t / D_h)$. The function depends on the value of Reynolds number which is based on the flow conditions. Where Reynolds number is < 2320, the flow is laminar and the friction factor is given by Equation (6) derived from the Hagen-Poiseuille model with a parabolic velocity distribution across the flow section. This model, which can be confirmed experimentally, does not depend on the relative roughness of the airway.

For Re > 2320, the flow is turbulent, and depending on the relative thickness of the fluid boundary layer δ (m) next to the wall, three cases are considered. The boundary layer is defined by the distance from the wall, from where the velocity is zero at the wall, to where the mean value of flow occurs; this mean value is almost constant in the core of the stream.

When the layer thickness is much larger than the wall roughness the flow falls in the smooth zone which is limited by the criteria $\varepsilon_t \cdot \mathrm{Re} < 65$. The friction factor is calculated by Equation (7), from Blasius, or Equation (8), from Nikuradse, depending on the value of Reynolds number. For a layer thickness similar to the wall roughness, the Colebrook's relationship, Equation (9) is used, and for a layer smaller than the wall roughness the flow is in the wholly rough zone. In this case, one can use the simplified Colebrook's relationship, Equation (10) for large Reynolds numbers with limiting criteria at the lower end of the Reynolds number given by $\varepsilon_t \cdot \mathrm{Re} > 1300$ (Dubbel, 1990). Colebrook's relationship, Equation (9) is an implicit function to determine the friction factor and an iterative method must be used in its calculation. An alternative approximate explicit formula to calculate the friction factor is given in Equation (11).

The relation between the Atkinson factor, K_{At}, as commonly used in the mine ventilation and the friction factor defined here is the following (Hartman, 1982):

$$K_{At} = \frac{f_t}{8} \rho_a \tag{12}$$

Figure 1 is a plot showing the friction factor as a function of the Reynolds number and different relative roughnesses (ε_t), this is known as the Moody diagram. The lines for Re > 2320 corresponds to turbulent flow. The lines to the left indicate lower values and laminar flow. The segmented curves in the turbulent region define the limits of its three zones, the area below the lower left curve correspond to the smooth zone, between both curves correspond to the transitional, and the area to the right of the upper curve is the wholly rough zone. The Blasius and Nikuradse's relationships are in the smooth zone and are represented by the heavy segmented lines in the diagram. When the Reynolds number is in the range 2000 < Re < 4000, the flow is unstable and should be avoided, however where pressure drop estimation is required, the friction factor of the turbulent region should be used. For the zone Between the transition and the smooth curves of Blasius

Laminar flow		Re < 2320	$f_t = \frac{64}{\mathrm{Re}}$	(Hagen-Poiseuille)	(6)
Turbulent Flow		Re > 2320			
Smooth Zone	$\frac{\delta}{e_t} >> 1$	$\varepsilon_t \cdot \mathrm{Re} < 65$			
		$2320 < \mathrm{Re} < 10^5$	$f_t = \frac{0.3164}{\mathrm{Re}^{0.25}}$	(Blasius)	(7)
		$10^5 < \mathrm{Re} < 10^8$	$f_t = 0.0032 + \frac{0.221}{\mathrm{Re}^{0.237}}$	(Nikuradse)	(8)
Transitional Zone	$\frac{\delta}{e_t} \approx 1$	$65 < \varepsilon_t \cdot \mathrm{Re} < 1300$			
			$\sqrt{f_t} = \frac{1}{2\log\left(\frac{\varepsilon_t}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f_t}}\right)}$	(Colebrook)	(9)
		Explicit Approximation	$f_t = \frac{1}{\left(2\log\left(\frac{\varepsilon_t}{3.715} + \frac{5.72}{\mathrm{Re}^{0.9}}\right)\right)^2}$		(11)
Wholly Rough Zone	$\frac{\delta}{e_t} < 1$	$\varepsilon_t \cdot \mathrm{Re} > 1300$			
			$f_t = \frac{1}{\left(2\log\left(\frac{3.7}{\varepsilon_t}\right)\right)^2}$	(Colebrook)	(10)

and Nikuradse, relations (9) or (11) should be used in order to have consideration of the surface roughness; the zone limiting curves are for theoretical reference purpose only.

For ventilation purposes the air density can be considered constant along the stream, and only a correction for altitude and temperature must be applied. To calculate the pressure at the elevation of the mine the following approximate relation can be used (h in km):

$$p_h = p_o 0.9^h \qquad (13)$$

where h = elevation (km)

To obtain the density of the air at the elevation h and air temperature T_a at the mine conditions, the following relation for a perfect gas can be applied:

$$\rho_a = \rho_o \left(\frac{p_h}{p_o} \frac{T_o}{T_a} \right) \qquad (14)$$

The air viscosity is practically independent of the air pressure in this application and depends mainly on the temperature T_a and can be calculated from Equation (15). The unit is (Pa·s), it includes the parameters T_e and C_e, and its use is restricted in the indicated ranges, however these are far beyond normal ventilation applications:

$$\mu_a = \frac{23.36 \cdot 10^{-6}}{1 + \frac{C_e}{T_a}} \sqrt{\frac{T_a}{T_e}} \qquad (15)$$

where $p < 737.6$ kPa
$233 \text{ K} < T_a < 573$ K, and
$T_e = 273.16$ K
$C_e = 100$ K

3 Determination of the absolute roughness of tunnels and shafts

Pressure drop measurements were performed on 25 straight and obstructions free sections of ventilation tunnels at the El Teniente mine of Codelco Chile. The tunnels ranged in size from 4.1 m x 4.0 m to 8.0 m x 7.5 m and shafts ranged from 1.5 to 5.0 m Ø, and the different wall finishes allowed for the determination of the airway's absolute equivalent roughness. The construction method

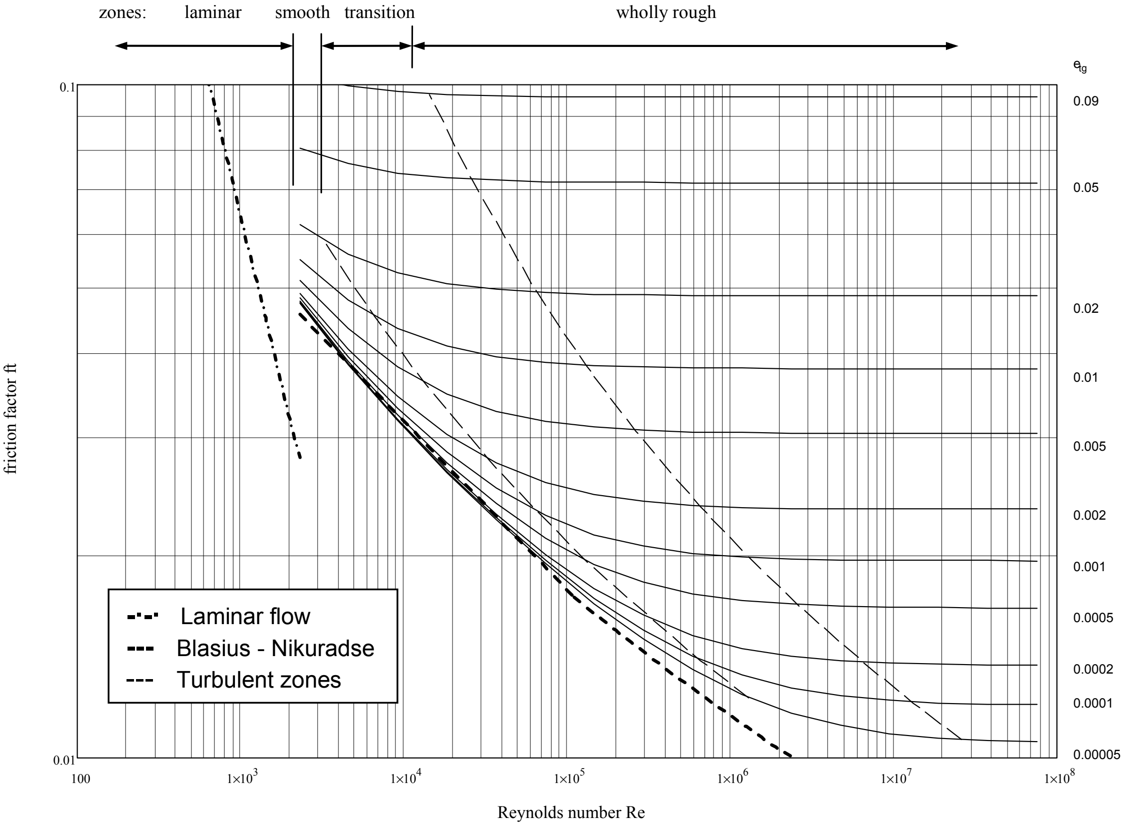

Figure 1 Friction factor against Reynolds number and roughness.

and the type of wall finishing were documented in order to correlate the calculated absolute equivalent roughness.

The area and perimeter of the cross section of the tunnels were measured using a photo profile taken with a flash that projects a radial beam to demark the section and a reference square of known area. Several pictures were taken along each tunnel and the average of the areas was used as representative for each airway type calculations.

The air speed was measured with a turbine type anemometer by sweeping it through a predefined path covering the whole cross sectional area. A velocity was then calculated using the measured time of the anemometer sweep. This was done several times at the same location in the tunnel; the average value was then used in further calculations. The pressure drops were measured at the same time as the air speed using the gauge-and-tube method and a Dwyer type inclined manometer which had a precision of 1 Pa.

In the case of the shafts and/or raises, only the nominal dimensions were available to determine the area and perimeter. The airflow was measured entering the raise and the differential pressure was measured with the gauge-and-tube method between two levels. The losses due to flow splitting, deflection and contraction at the inlet; and expansion, deflection and joining at the outlet of the shaft were included in the pressure measurements. They could not be separated. So the equivalent roughness obtained for vertical airways include these losses. This explains their higher absolute roughness compared to similar surface finishes which are in the range of 400-500 mm in horizontal airways.

Air temperature, humidity and atmospheric pressure were measured in each case to determine the air density. Table 1 shows the absolute roughness obtained according to the excavation method, wall finishing and reinforcement.

All tunnels and shafts were excavated by a drill and blast method except for the last shaft which was made with a raise borer. The floor finishing of the tunnels is the result of the excavation method and no extra finishing was done. The evenness of the floor describes the quality of the surface obtained but does not always coincide with the description of the walls and roof.

The area considered for tunnels with steel frames correspond to the free section inside the frames. The area deviation gives an idea of the uniformity of the section

Table 1 Absolute roughness according to surface finish

Type of excavation and use	Wall finishing	Characteristics of walls and roof	Floor evenness	Area deviation %	Absolute roughness e_t (mm)
Intake Adit	rock surface without bolts	medium roughness	Uneven	1	318
		high roughness	Uneven	5	459
Exhaust Adit	rock surface with bolts	smoothened by dust	Uneven	1	206
Intake Adit	rock surface with bolts & mesh	high roughness	Even	3	554
Exhaust Adits	rock surface with bolts & mesh	smoothened by dust	Even	2	337
		medium roughness	uneven	5	426
		high roughness	even	15	509
Intake Adit	shotcrete	medium roughness	even	-	130
		high roughness	even	6	467
Return Adit	shotcrete	low roughness, smoothened by dust	uneven	-	176
		low roughness, smoothened by dust	even	3	259
		high roughness	very uneven	7	261
Intake Adits	steel frames	spaced at 1 m, protruding 500 mm	uneven	5	305
Return Adit	steel frames	spaced at 1 m, protruding 500 mm	even	-	608
			uneven	1	675
Intake Adits	steel frames with timber lining	lining flushed with flange	even	4	135
Return Adits	steel frames with timber lining	lining flushed with flange	even	7	114
Return Adits	concrete reinforcement, complete	low roughness	even	-	82
			even	6	22
Intake shaft	rock surface with bolts & mesh	round section	-	-	928
Return shaft	rock surface with bolts & mesh	round section with ladder	-	-	976
Intake shaft	Smooth rock (raise borer)	round section	-	-	13

along the tunnel. It can be observed that the absolute roughness increases with increasing area deviation which compensates for the increased pressure drop due to the portions of the tunnel having smaller sections.

In some of the return tunnels a dust deposit was observed, which had built up during the operation period, and had some smoothening effect by filling the voids of the surface as can be seen when comparing the roughness of the airways. So when an exhaust tunnel is recently excavated a higher roughness should be used for ventilation pressure drop estimates than when it has been in operation for a longer time.

4 Calculation examples

The following examples are given to show how to use the equations presented in this paper to calculate pressure drop along a tunnel.

Case 1: An intake ventilation tunnel excavated by drill and blast method with bare rock surface, reinforcing bolts and mesh, of 4 m width by 4 m height and a circular roof of 2 m radius, located at an elevation of 2200 m above sea level conducts air of 358C at a speed of 12 m/s. Calculate the pressure drop per unit length.

Solution: From the surface finish of the tunnel the absolute roughness according to Table 1 is e_t = 554 mm, and considering the section dimensions the area and perimeter are: A_t=14.28 m^2, P_t=14.28 m. Using Equation (3) the hydraulic diameter is D_h = 4 m and the relative roughness from Equation (5) is ε_t = 0.1385. The atmospheric pressure at the elevation of the tunnel from Equation (13) p_h is 80.4 kPa, using Equation (14) the air density ρ_a is 0.955 kg/m^3 and with Equation (15) the air viscosity μ_a is 2.004·10^{-6} Pa·s. Using Equation (4) the Reynolds number is calculated as Re=2.29·10^6 and since $\varepsilon_t \cdot \mathrm{Re}$=3.17·10^5 the flow falls in the wholly rough zone (see Figure 1).

In this case Equation (10) must be used to determine the friction factor becoming f_t = 0.1227. Through Equation (1) the pressure drop per unit length becomes Δp = 2.109 Pa/m. The Atkinson friction factor from Equation (12) K_{At} is 0.0146 kg/m^3.

Case 2: Another tunnel with the same characteristics as before, that is the same absolute roughness, elevation, air temperature and air speed but with the following dimensions 5.5 m width, 5.5 m height and 2.75 m roof radius. Obtain the pressure drop per unit length.

Solution: Similarly, as before the area and perimeter results are: A_t=27.004 m^2, P_t=19.64 m, the hydraulic diameter is D_h =5.5 m, in this instance and the relative roughness becomes ε_t =0.1007. The Reynolds number is Re=3.14·10^6 and again it is in the wholly rough zone so resulting in f_t=0.1020 and the pressure drop per unit length Δp = 1.275 Pa/m and the Atkinson friction factor K_{At} is 0.0122 kg/m^3.

If the Atkinson friction factor derived for the smaller airway had been used in a simulation for this larger airway, the pressure drop would have been Δp = 1.534 Pa/m which is 20% higher than that obtained assuming the same roughness. This is due to the scale factor which is not accounted for in typical calculations of friction factor.

5 Conclusions

The intention of this paper is to encourage ventilation engineers to move to the modern fluid mechanics calculation procedures, for which ample theoretical background exist, to achieve more accurate pressure drop estimations. Existing computational software can be easily adapted to this method, having in mind that the friction factors of the tunnels do not vary significantly considering that normally they operate in the wholly rough zone depending mainly on the relative roughness. By adding a subroutine in each iteration step in the software for the determination of the flow in each branch, the friction factor can to be calculated and obtained by determining the air velocity and thus the Reynolds number. Only in secondary branches in the mine ventilation network, may laminar flow occur.

The analysis and results presented in this paper show the importance of:

- Firstly, being aware that Atkinson friction factors K_{At} given in standard ventilation texts need to be considered in relation to the original size of the airways where they were determined. Typically, dimensional information is not available so whether they are appropriate for modern mechanized mining methods and generally larger dimension airways needs to be considered.
- Secondly, this work reinforces the need to determine friction factors in situ. When determined in place, subsequent analyses could still be used to simplified the Atkinson approach as long as the conditions remain similar, i.e. the same general size and turbulence condition.
- Thirdly, consideration also has to be given to how the condition of the surface may change with time. This study has shown that dust accumulation can have a smoothing effect that might need to be taken into account. Conversely there is the potential of an event reversing dust accumulation (such as nearby blasting) that could revert the friction factor to its original state.

References

Blevins, R.D, PhD. (1984). *Applied Fluid Mechanics Handbook*, ISBN 0-442-21296-8

Dubbel. (1990). *Taschenbuch für den Maschinenbau*. 17 Auslage, Springer, ISBN 3-540-52381-2

Hartman, H.L. (1982). *Mine Ventilation and Air Conditioning*. Second Edition, Kreiger Publishing, ISBN0894644718.

McPherson, M.J. (2007). *Subsurface Ventilation and Environmental Engineering*. Second Edition, Springer, ISBN 0412353008